



MATHEMATICS SPECIALIST : UNITS 1 & 2, 2023

Test 2 – Vectors; Circle Properties (10%)

1.3.1 – 1.3.9, 1.3.14, 1.1.17; 1.1.6 – 1.1.15

PM

Time Allowed 20 minutes	First Name	Surname	Marks
			24 marks

Circle your Teacher's Name: Ms Chua Mr Koulianos Mr Liu Ms Robinson

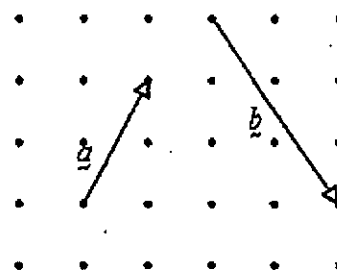
Assessment Conditions: (N.B. Sufficient working out must be shown to gain full marks)

- ❖ Calculators: Not Allowed
- ❖ Formula Sheet: Provided
- ❖ Notes: Not Allowed

PART A – CALCULATOR FREE

1. (1, 2, 2, = 5 marks)

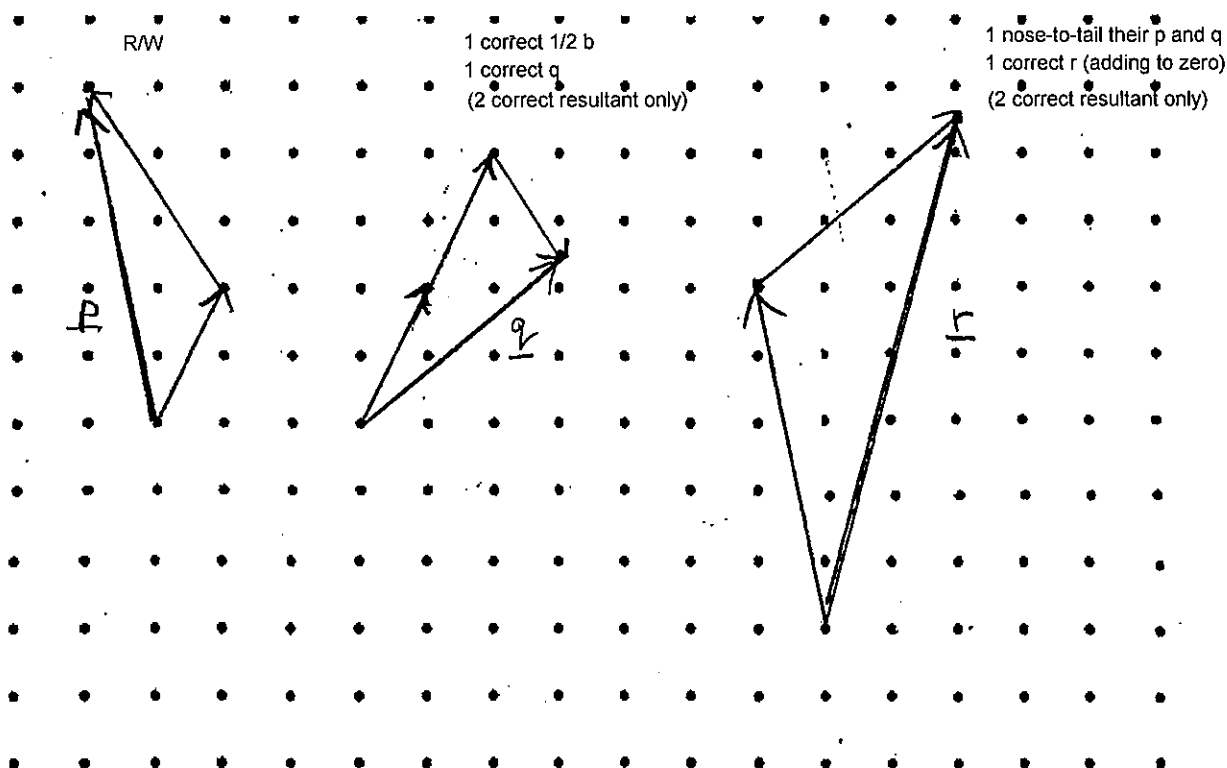
With a and b as defined in the diagram on the right,
draw each of the following on the grid below
clearly labelling each vector.



(a) $p = a - b$

(b) $q = 2a + \frac{1}{2}b$

(c) the vector r such that $p + q + r = 0$



2. (2, 2, 4 = 8 marks)

Given $p = 2i + 3j$, $q = -12i - 5j$ and $r = xi + 3j$ determine each of the following:

(a) $-p + 2q = - (2i + 3j) + 2(-12i - 5j)$
 $= -26i - 13j$

1 suitable working
 1 ans
 (-1 per error)

(b) A vector in the opposite direction as q , but with three times the magnitude of vector p

$|p| = \sqrt{13}$
 $-q = 12i + 5j$
 $|q| = 13$

Required vector
 $= \frac{3\sqrt{13}}{13} (12i + 5j)$

1 correct magnitude
 1 correct direction

(c) The value of x such that $|3p - r| = 10$

$\left| 3 \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} x \\ 3 \end{pmatrix} \right| = 10$

$\left| \begin{pmatrix} 6-x \\ 6 \end{pmatrix} \right| = 10$

$(6-x)^2 + 6^2 = 10^2$

$(6-x)^2 = 8^2$

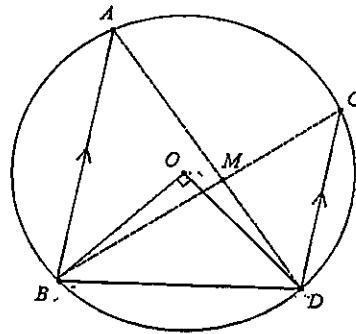
$6-x = \pm 8$

$\Rightarrow x = -2 \text{ or } x = 14$

1 set up equation
 2 simplifies correctly (-1 per error)
 1 finds both solutions

3. (2, 3, 2 = 7 marks)

In the circle on the right, A, B, C and D lie on the circumference of a circle with centre O .



(a) Prove that $\angle BAD = \angle BCD = 45^\circ$.

$\angle BAD = 45^\circ$ (Angle at circumference is $\frac{1}{2}$ angle at centre)
 1 uses angle at the centre
 1 completes the proof

$\angle BCD = 45^\circ$ (Angles in same arc)

$\therefore \angle BAD = \angle BCD = 45^\circ$

(b) Prove that $AD \perp BC$

$\angle BAD = \angle ADC = 45^\circ$

(Alt. angles in parallel lines)

As $\angle MCD = \angle MDC = 45^\circ$

1 uses alternate angles
 1 recognises isosceles triangle
 1 completes the proof

$\Rightarrow \triangle MCD$ is isosceles

$\therefore \angle CMD = 90^\circ \Rightarrow AD \perp BC$

(c) Prove that M lies on the circle BOD .

$\angle BOD = 90^\circ$

(Given)

BD is diameter of circle passing through BOD

(Angle in a semi-circle)

$\angle BMD = 90^\circ$

(Angle in same arc as $\angle BOD$)

$\therefore M$ lies on circle BOD

1 uses centre in semi-circle to explain BD is a diameter.
 1 Uses angle in the same arc to explain why M is on the circle.



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Test 2 – Vectors; Circle Properties (10%)

1.3.1 – 1.3.9, 1.3.14, 1.1.17; 1.1.6 – 1.1.15

PM

Time Allowed 30 minutes	First Name	Surname	Marks
			36 marks

Circle your Teacher's Name: Ms Chua Mr Koulianos Mr Liu Ms Robinson

Assessment Conditions: (N.B. Sufficient working out must be shown to gain full marks)

- ❖ Calculators: Allowed
- ❖ Formula Sheet: Provided
- ❖ Notes: Not Allowed

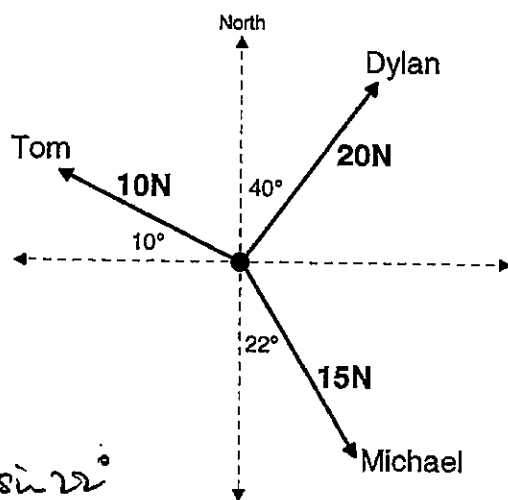
PART B – CALCULATOR ASSUMED

4 (7 marks)

Robert is asleep in his chair and unbeknown to him Tom, Dylan and Michael have each tied ropes around him and begin to pull with the forces and directions shown.

In which direction and with what force does the sleeping Robert move?

Give the direction as a bearing to the nearest degree.



- 2 correctly sets up eqn and finds hor component (-1 per error)
- 1 correctly calculates vert component
- 1 uses hor & vert components to calc the mag of total force

$$\text{Horiz} : 20 \sin 40^\circ - 10 \cos 10^\circ + 15 \sin 22^\circ$$

$$\approx 8.627$$

$$\text{Vert} : 20 \cos 40^\circ + 10 \sin 10^\circ - 15 \cos 22^\circ$$

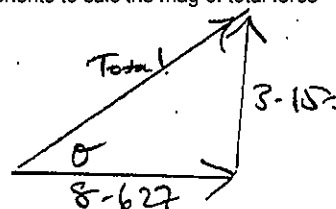
$$\approx 3.150$$

$$\text{Mag. Total force} = \sqrt{8.627^2 + 3.150^2} = 9.184$$

$$\theta = \tan^{-1} \left(\frac{3.150}{8.627} \right)$$

$$= 20.1^\circ$$

- 1 correctly calculates theta
- 1 uses theta to find the bearing
- 1 States force with units

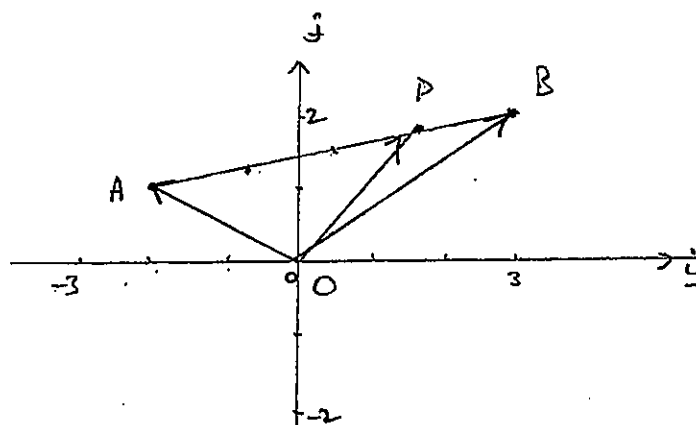


Direction 070° as a bearing (to nearest degree)
with a force of 9.2N.

5. (3 marks)

If points A and B have position vectors $-2\mathbf{i} + \mathbf{j}$ and $3\mathbf{i} + 2\mathbf{j}$ respectively, find the position vector of point P such that $\overrightarrow{AP} : \overrightarrow{PB} = 3:1$

$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{OA} + \frac{3}{4} \overrightarrow{AB} \\ &= \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \frac{3}{4} \begin{pmatrix} 5 \\ 1 \end{pmatrix} \\ &= \frac{7}{4} \mathbf{i} + \frac{7}{4} \mathbf{j}\end{aligned}$$



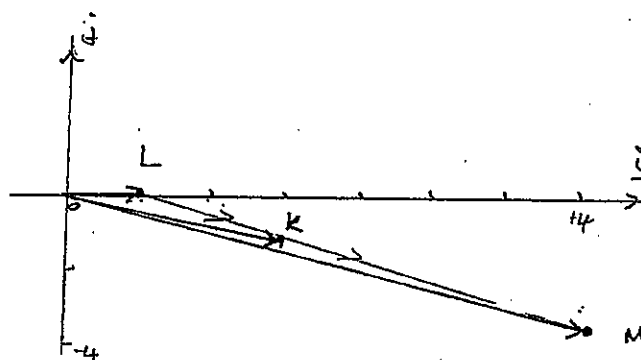
2 set s up an appropriate vector eqn (-1 per error)
1 correct ans

6. (4 Marks)

Points K, L and M have position vectors $6\mathbf{i} - \mathbf{j}$, $2\mathbf{i}$ and $14\mathbf{i} - 3\mathbf{j}$ respectively.

Use vectors to prove that K, L and M are collinear.

$$\begin{aligned}\overrightarrow{LK} &= 6\mathbf{i} - \mathbf{j} - 2\mathbf{i} \\ &= 4\mathbf{i} - \mathbf{j} \\ \overrightarrow{LM} &= 14\mathbf{i} - 3\mathbf{j} - 2\mathbf{i} \\ &= 12\mathbf{i} - 3\mathbf{j} \\ &= 3(4\mathbf{i} - \mathbf{j})\end{aligned}$$



$\therefore$ Points K, L and M are collinear as $\overrightarrow{LK}$ and $\overrightarrow{LM}$ are parallel with the point L in common

1 finds LK
1 finds LM
1 states LK and LM are scalar mult so parallel
1 states they have a point L in common

NB: May use different vectors but must show/have a point in common)

7. (6,1 = 7 marks)

A plane needs to travel directly from City A (0, 0) km to City B (-600, 800) km.

The plane travels at a speed of 75 km/h in still air. The wind's velocity is $-50\mathbf{i} + 20\mathbf{j}$ km/h.

The $\mathbf{j}$ vector faces due North and the $\mathbf{i}$ vector faces due East.

- (a) Determine the direction, as a bearing, that the plane should travel towards to fly directly from City A to City B.

Let $t \equiv$ time taken in hours

$$t \left[\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} -50 \\ 20 \end{pmatrix} \right] = \begin{pmatrix} -600 \\ 800 \end{pmatrix}$$

- 1 sets up suitable vector eqn (from their diagram)
- 2 creates 2 eqns to solve simultaneously
- 1 solves for a & b discarding a 2nd soln
- 1 calculates theta
- 1 states correct bearing

$$\begin{aligned} \therefore t(a-50) &= -600 \\ t(b+20) &= 800 \end{aligned}$$

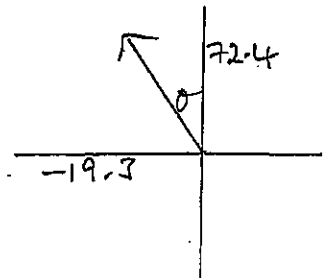
By division, $\frac{a-50}{b+20} = \frac{-6}{8} = \frac{-3}{4} \quad \text{--- (1)}$ Also, $a^2 + b^2 = 75^2 \quad \text{--- (2)}$
(from speed of wind)

Solve (1) & (2) simultaneously by CAS $\Rightarrow a = -19.3$, $b = +72.4$
(Discard 2nd solⁿ - wrong direction)

$$\theta = \tan^{-1} \left(\frac{19.3}{72.4} \right) = 14.9^\circ$$

$$\begin{aligned} \therefore \text{Bearing} &= 360^\circ - 14.9^\circ \\ &= 345.1^\circ \end{aligned}$$

(or 345° to nearest degree)



- (b) How long does the flight take (to the nearest minute)?

$$\begin{aligned} \text{Sub } a = -19.3 &\Rightarrow t(-19.3 - 50) = -600 \\ &\Rightarrow t \approx 8.66 \text{ h} \\ &= 8 \text{ h } 39 \text{ min} \\ &(\text{or } 519 \text{ minutes}) \end{aligned}$$

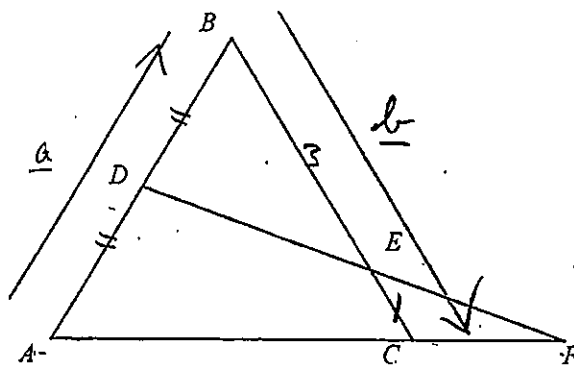
R/W (FT from their (a))
Must be to the nearest minute.

8. (2, 5, 2 = 9 marks)

D is a point in $\overline{AB}$ such that $\overline{AD} = \frac{1}{2}\overline{AB}$

E is a point in $\overline{BC}$ such that $\overline{BE} = \frac{3}{4}\overline{BC}$

Let $\overline{AB} = a$ and $\overline{BC} = b$



(a) Find expressions for the following vectors in terms of a and/or b

(i) $\overline{AC} = \underline{a} + \underline{b}$ R/W

(ii) $\overline{DE} = \frac{1}{2}\underline{a} + \frac{3}{4}\underline{b}$ R/W

(b) By letting $\overline{DF} = h\overline{DE}$ and $\overline{AF} = k\overline{AC}$, use the fact that $\overline{AD} + \overline{DF} = \overline{AF}$ to find the values of h and k .

$$\begin{aligned}\overrightarrow{AD} + \overrightarrow{DF} &= \overrightarrow{AF} \\ \frac{1}{2}\underline{a} + h\left(\frac{1}{2}\underline{a} + \frac{3}{4}\underline{b}\right) &= k(\underline{a} + \underline{b}) \\ \frac{1}{2}\underline{a} + \frac{h}{2}\underline{a} + \frac{3h}{4}\underline{b} &= k\underline{a} + k\underline{b} \\ \left(\frac{1}{2} + \frac{h}{2} - k\right)\underline{a} &= \left(k - \frac{3}{4}h\right)\underline{b}\end{aligned}$$

1 substitutes into given eqn
1 expands and rearranges into
ha = kb form
1 creates 2 eqns to solve
1 finds h
1 finds k

$$\begin{aligned}\text{As } \underline{a} \nparallel \underline{b}, \quad \frac{1}{2} + \frac{h}{2} - k &= 0 \\ \frac{1}{2} + \frac{h}{2} &= k \quad \text{--- ①} \\ k - \frac{3}{4}h &= 0 \\ \frac{3}{4}h &= k \quad \text{--- ②}\end{aligned}$$

$$\begin{aligned}\therefore \frac{1}{2} + \frac{h}{2} &= \frac{3}{4}h \quad \text{from ① + ②} \\ \Rightarrow h &= 2 \quad \text{and } k = 1.5\end{aligned}$$

(c) If $|\overline{AC}| = 12$, what is $|\overline{CF}|$?

$$\begin{aligned}|\overrightarrow{AF}| &= 1.5 |\overrightarrow{AC}| \\ &= 1.5 \times 12 \\ &= 18\end{aligned}$$

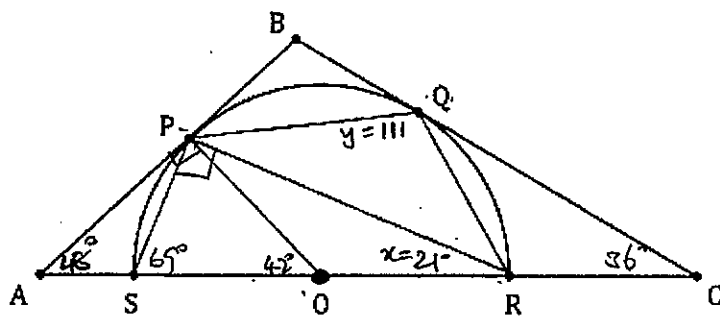
1 finds mag AF (FT from their k)
1 finds mag CF

$$\therefore \underline{|\overline{CF}| = 6}$$

9. (6 marks)

The diagram shows a semi-circle, with diameter SR and centre O, circumscribed by triangle ABC, in which $\angle BAC = 48^\circ$ and $\angle BCA = 36^\circ$.

Determine, with reasons, the size of angles $\angle PRO$ and $\angle PQR$.



$$\angle AOP = 90^\circ - 48^\circ = 42^\circ$$

($\angle APO = 90^\circ$ tan-radius)

$$\angle PRO = 42^\circ \div 2 = 21^\circ$$

(Centre $\angle = 2 \times$ circ $\angle$)

$$\angle PSR = 90^\circ - 21^\circ = 69^\circ$$

(as $\angle SPR = 90^\circ$ angle in semi circle)

$$\angle PQR = 180^\circ - 69^\circ = 111^\circ$$

(Opp's PQRS cyclic Quad.)

1 finds angle AOP

1 finds angle PRO

1 reasoning

1 angle PSR

1 angle PQR

1 reasoning

Note: Students may use alternate reasoning.